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Tugas Nilai Maks Min Fungsi

25 Dapatkan semua titik kritis dan tunjukkan apakah titik itu memberikan maksimum, minimum lokal, atau titik pelana

c). $F(x,y) = e^{-(x^2+y^2-4y)} + (y-2)^2$

→ $F_x = 0$

$$-2x\underbrace{e^{-(x^2+y^2-4y)}}_{\text{Pasti bilangan positif } (\neq 0)} = 0$$

maka $-2x$ haruslah 0

※ $x = 0$

→ $F_y = 0$

$$-(2y-4)e^{-(x^2+y^2-4y)} + 2(y-2) = 0$$

$$\left[1 - e^{-(x^2+y^2-4y)}\right](2y-4) = 0$$

※ $y = 2$

$$1 - e^{-(x^2+y^2-4y)} = 0$$

$$e^{-(x^2+y^2-4y)} = e^0$$

$$-x^2 - y^2 + 4y = 0$$

→ Substitusi $x = 0$. $-(0)^2 - y(y-4) = 0$

※ $y = 0$ V ※ $y = 4$

Jadi, terdapat 3 calon titik ekstrem, yaitu $(0,0), (0,2), (0,4)$

▶ Uji pada turunan parsial kedua, termasuk jenis ekstrem apa.

→ $F_{xx} = -2e^{-(x^2+y^2-4y)} + \left(-2x\left(-2x e^{-(x^2+y^2-4y)}\right)\right)$

$= -2e^{-(x^2+y^2-4y)} + 4x^2 e^{-(x^2+y^2-4y)}$

$= e^{-(x^2+y^2-4y)}\left[4x^2 - 2\right]$

→ $F_{yy} = -2\left[e^{-(x^2+y^2-4y)}\right] + \left((-2y+4)[-2y+4]e^{-(x^2+y^2-4y)}\right) + 2$

$= -2\left[e^{-(x^2+y^2-4y)}\right] + (-2y+4)^2 e^{-(x^2+y^2-4y)} + 2$

$= \left[e^{-(x^2+y^2-4y)}\right]\left(-2 + (-2y+4)^2\right) + 2$

→ $F_{xy} = 0\left[e^{-(x^2+y^2-4y)}\right] + (-2x)(-2y+4)\,e^{-(x^2+y^2-4y)}$

$= 2x(2y-4)e^{-(x^2+y^2-4y)}$

※ $\Delta = f_{xx}(x_0,y_0)\cdot f_{yy}(x_0,y_0) - \left[f_{xy}(x_0,y_0)\right]^2$

$= e^{-(x^2+y^2-4y)}\left[4x^2-2\right]\left(\left[e^{-(x^2+y^2-4y)}\right](-2+(-2y+4)^2) + 2\right) - \left[2x(2y-4)e^{-(x^2+y^2-4y)}\right]$

→ Uji di titik $(0,0)$

$\Delta = e^0[-2]\left([e^0](-2+16) + 2\right) - \left[2(0)(-4)e^0\right]$

$= (-2\cdot 1[14] + 2) - 0$

$= (-26)$

Karena $\Delta < 0$, maka titik tersebut merupakan titik pelana

→ Uji di titik $(0,2)$

$\Delta = e^4[-2]\left(e^4(-2)\right) + 2 - \left[2(0)(0)\cdot e^4\right]$

$= 4e^8 + 2$

$f_{xx} = e^4[-2]$

$= -2e^4$

Karena $\Delta > 0$ dan $f_{xx} < 0$, maka titik tersebut merupakan titik ekstrem maksimum

→ Uji di titik $(0,4)$

$\Delta = e^0[-2]\left([e^0](-2+16) + 2\right) - \left[0(4)e^0\right]$

$= -2[14+2]$

$= -32$

Karena $\Delta < 0$, maka titik tersebut merupakan titik pelana

Jadi, Ketiga titik tersebut setelah dianalisis termasuk dalam :

- $(0,0) \longrightarrow$ Titik Pelana
- $(0,2) \longrightarrow$ Titik Ekstrem Maksimum
- $(0,4) \longrightarrow$ Titik Pelana

d). $f(x,y) = xy + \frac{2}{x} + \frac{4}{y}$

$\Rightarrow f_x = 0$

$y + 2(-1).\frac{1}{x^2} = 0$

$y - \frac{2}{x^2} = 0 \; .... (1)$

$\Rightarrow f_y = 0$

$x - \frac{4}{y^2} = 0$

$x = \frac{4}{y^2} \; .... (2)$

$\blacktriangleright \; y - \frac{2}{(4/y^2)^2} = 0$

$y = \frac{2}{16} . y^4$

$\frac{1}{8}y^4 - y = 0$

$y\left[\frac{1}{8}y^3 - 1\right] = 0$

$y = 0 \lor \frac{1}{8}y^3 - 1 = 0$

TM, karena akan ada penyebut = 0

$y^3 = 8$

$y = 2$

$x = \frac{4}{y^2} = \frac{4}{4} = 1$

$y = \frac{2}{x^2} \rightarrow x^2 = 1 \rightarrow x = \pm 1$

Berarti, kemungkinan titik ekstremnya :

- $(1,0), (-1,0) \Rightarrow$ TM ; • $(1,2), (-1,2)$ ← TM, tidak memenuhi salah satu persamaan

$\longrightarrow$ Uji turunan kedua untuk cek jenis ekstremnya

$\Rightarrow f_{xx} = -2.(-2)\frac{1}{x^3} = \frac{4}{x^3}$

$\Rightarrow f_{yy} = 0 - 4.(-2).\frac{1}{y^3} = \frac{8}{y^3}$

$\Rightarrow f_{xy} = 0$

$\Delta = f_{xx}(x_0,y_0).f_{yy}(x_0,y_0) - [f_{xy}(x_0,y_0)]^2$

$= \frac{4}{x^3}.\frac{8}{y^3} - [0]^2$

$= \frac{32}{x^3y^3}$

$\Rightarrow$ Uji di titik $(1,2)$

$\Delta = \frac{32}{8} = 4 \; (+)$

$f_{xx}(1,2) = 4 \; (+)$

Karena $\Delta > 0$ dan $f_{xx}(x_0;y_0) > 0$ maka termasuk nilai ekstrem Minimum lokal

Titik $\Rightarrow f(1,2) = (1)(2) + \frac{2}{(1)} + \frac{4}{(2)}$

$= 2 + 2 + 2$

$= 6 \Rightarrow$ Titiknya $= (1,2,6)$

28. Dapatkan Nilai Ekstrim

b). $x^3 - y^2 + z^2 - 3x + 4y + z - 8 = 0$

$\Rightarrow F_x = 0$ | $\Rightarrow F_y = 0$ | $F_{xx} = 6x$ | $F_{xy} = 0$

$3x^2 - 3 = 0$ | $-2y + 4 = 0$ | $F_{yy} = -2$ | $F_z = 2z+1$

$3(x^2-1) = 0$ | $y = 2$ | $F_{zz} = 2$

$3(x+1)(x-1) = 0$

$x = -1 \lor x = 1$

▶ Substitusi ke persamaan awal untuk ketemu titik kritisnya

$(1,2) \rightarrow (1)^3 - (2)^2 + z^2 - 3(1) + 4(2) + z - 8 = 0$

$= 1 - 4 + z^2 - 3 + 8 + z - 8 = 0$

$z^2 + z - 6 = 0$

$(z+3)(z-2) = 0$

$z = -3 \lor z = 2 \Rightarrow$ Titiknya $(1,2,2), (1,2,-3)$

$(-1,2) \rightarrow (-1)^3 - (2)^2 + z^2 - 3(-1) + 4(2) + z - 8 = 0$

$-1 - 4 + z^2 + 3 + 8 + z - 8 = 0$

$z^2 + z - 2 = 0$

$(z+2)(z-1) = 0$

$z = -2 \lor z = 1 \Rightarrow$ Titiknya $(-1,2,-2), (-1,2,1)$

Jadi, titik-titik kritisnya : $(1,2,2), (1,2,-3), (-1,2,-2), (-1,2,1)$

▶ Tentukan Δ untuk menentukan jenis ekstrem

$\Delta = (f_{xx} f_{yy} - [f_{xy}]^2) = -12x - [0]^2$

$\Rightarrow$ Di $(1,2,2)$

$\Delta = f_{xx} f_{yy} - (f_{xy})^2$

$= -12x$

$= -12$

$\rightarrow$ Titik Sadel / pelana

$\Rightarrow$ Di $(1,2,-3)$

$\Delta = f_{xx} f_{yy} - (f_{xy})^2$

$= -12x$

$= -12$

$\rightarrow$ Tidak Sadel / pelana

$\Rightarrow$ Di $(-1,2,-2)$

$\Delta = f_{xx} f_{yy} - (f_{xy})^2$

$= -12(-1)$

$= 12$

$z_{xx} = -\dfrac{F_{xx}}{F_z} = -\dfrac{6x}{2z+1}$

$= -\dfrac{6(-1)}{2(-2)+1}$

$= -2$ (maksimum)

$\Rightarrow$ Di $(-1,2,1)$

$\Delta = f_{xx} f_{yy} - (f_{xy})^2$

$= -12(-1)$

$= 12$

$z_{xx} = -\dfrac{F_{xx}}{F_z} = -\dfrac{6x}{2z+1}$

$= -\dfrac{6(-1)}{2(1)+1}$

$= \dfrac{6}{5}$ (minimum)

$\Rightarrow$ Nilai maksimum $\Rightarrow -2$

Nilai minimum $\Rightarrow 1$

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33. Jarak terpendek titik asal dan permukaan : $x^2y - z^2 - 9 = 0$

Jarak $\Rightarrow$ $d = \sqrt{x^2+y^2+z^2}$

$d^2 = x^2+y^2+z^2$

Persamaan : $z^2 = x^2y - 9$

$d^2 = x^2+y^2+x^2y-9$

$\Rightarrow F_x = 2x + 2xy = 0$

$= 2x(1+y) = 0$

$x = 0 \vee y = -1$

$\Rightarrow F_y = 2y + x^2 = 0$

Substitusi $x = 0 \rightarrow 2y = 0$,

$y = 0$

Substitusi $y = -1 \rightarrow x^2 - 2 = 0$

$x = \pm\sqrt{2}$

$\Rightarrow$ Titiknya $(0,0), (\sqrt{2},-1), (-\sqrt{2},-1)$

- $F_{xx} = 2 + 2y$
- $F_{yy} = 2$
- $F_{xy} = 2x$

$\Delta = f_{xx} f_{yy} - [f_{xy}]^2$

$= (2+2y)2 - 4x^2$

$= 4 + 4y - 4x^2$

$= 4[1 + y - x^2]$

$(0,0) \rightarrow \Delta = (2+2y)2 - [2x]^2$

$= 4$ (+)

$f_{xx} = 2+2y = 2$ (+)

$\rightarrow$ Minimum

$(\sqrt{2},-1) \rightarrow \Delta = 4[-2]$

$= -8$ (−)

$\rightarrow$ Titik Sadel / pelana

$(-\sqrt{2},-1) \rightarrow \Delta = 4[-2]$

$= -8$

$\rightarrow$ Titik Sadel / pelana

$z^2 = x^2y - 9$

$z^2 = -9$

$d = \sqrt{x^2+y^2+z^2}$

$= \sqrt{0-9}$

$= \sqrt{-9}$ (TM)

Jadi, belum ada jarak terdekat.

* Apabila persamaan diubah $\rightarrow x^2y - z^2 + 9 = 0$

$z^2 = x^2y + 9$

Saat $(0,0) \rightarrow z^2 = 9$

$d = \sqrt{x^2+y^2+z^2} = \sqrt{9}$

$= 3$